Charm Physics in a unified nonperturbative approach

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• Important hadronic effects stem from non-perturbative QCD contributions to strong \textit{and} weak decay amplitudes, whether in beauty and charm decays.

• Charm decays are particularly afflicted by hadronic uncertainties since corrections to the decay amplitudes of order $\Lambda_{\text{QCD}}/m_c$ are \textit{not} under control.

• Charm physics actively studied by LHCb, BaBar, Belle, BES, CLEO, FOCUS, RHIC ...

• PANDA at GSI will contribute to further development of \textit{charm} physics.

• Charmonium production rate at RHIC is sensitive to existence and properties of the intermediate “quark-gluon plasma” \textit{as well as} to final-state interactions ($J/\psi$ decays).
A few examples of hadronic charmed decays
Strong decay $D^* \rightarrow D\pi$

$$A(D^* \rightarrow D\pi) = \epsilon_{\mu D^*}^{\lambda_{D^*}} (p_{D^*}) M^\mu(p_D^2, p_{D^*}^2) := \epsilon_{\mu D^*}^{\lambda_{D^*}} (p_{D^*}) p_D^\mu g_{D^* D \pi}$$

$$M^\mu(p_D^2, p_{D^*}^2) = N_c \text{tr} \int_{-\Lambda}^\Lambda \frac{d^4k}{(2\pi)^4} \tilde{\Gamma}_D(k; -p_D) S_c(k + p_{D^*}) i\tilde{\Gamma}_{D^*}^\mu(k; p_{D^*}) S_u(k) \tilde{\Gamma}_\pi(k; -Q_{\pi}) S_u(k + Q_{\pi})$$

The coupling $g_{H^*H\pi}$ can be calculated even if decay is kinematically forbidden: $B^*$: $m_{B^*} - m_B = 46 \text{ MeV}$
Unphysical decay $B^* \to B\pi$ to extract effective heavy quark coupling

Heavy Quark Effective Theory

- Dynamics is constrained by heavy quark symmetry.
- Blind to the heavy quark flavor and spin.
- Heavy pseudoscalar and vector mesons are mass degenerate.
- Can be improved upon — take into account light degrees of freedom, chiral symmetry breaking

$\Rightarrow$ HMChPT.

\[ L_{\text{heavy}} = -\text{tr}_a \text{Tr}[\bar{H}_a i\nu \cdot D_{ba} H_b] + g \text{tr}_a \text{Tr}[\bar{H}_a H_b \gamma_\mu A_{ba}^{\mu} \gamma_5] \]

\[ D_{ba}^{\mu} H_b = \partial_\mu H_a - H_b \frac{1}{2} [\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger]_{ba} ; \]

\[ A_{ab}^{\mu} = \frac{i}{2} [\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger]_{ab} ; \]

\[ H_a(v) = \frac{1 + \frac{f_\pi}{f}}{2} \left[ P^{*a}_\mu (v) \gamma_\mu - P^a (v) \gamma_5 \right] ; \]

\[ \xi = \exp(i\Phi/f_\pi) ; \]

$\Phi$ is matrix of $N_f^2 - 1$ pseudo-Goldstone boson.

Generalization to higher spin structures

- $D^* \rightarrow D\pi$ can be generalized to three-point functions for vector mesons with off-shell momenta.
- Couplings $D\rho D, D^*\rho D, D^*\rho D^*$ needed in final-state interactions with intermediate $D - D^*$ states.

\[ D^*\rho D \text{ coupling: } \mathcal{M} \equiv \varepsilon^\rho_\alpha \varepsilon^{D^*}_\beta M^{\alpha\beta}(p^{D^*}, p^\rho) = \varepsilon^{\alpha\mu\nu} \varepsilon_\alpha(p^{D^*}) \varepsilon_\beta(p^\rho) p^\mu_\rho q^\nu_\nu \frac{g_{D^*\rho D}}{m_{D^*}} \]

\[ D^*\rho D^* \text{ coupling: } \\
M^{\mu\nu\rho}(p_1, p_2) = G_1 g^{\mu\nu} p^\rho_2 + G_2 g^{\mu\rho} p^\nu_2 + G_3 g^{\nu\rho} p^\mu_2 + G_4 g^{\mu\nu} p^\rho_1 + G_5 g^{\mu\rho} p^\nu_1 + G_6 g^{\nu\rho} p^\mu_1 + G_7 p^\mu_2 p^\nu_2 p^\rho_2 \\
+ G_8 p^\mu_1 p^\nu_2 p^\rho_2 + G_9 p^\mu_2 p^\nu_1 p^\rho_2 + G_{10} p^\mu_2 p^\nu_2 p^\rho_1 + G_{11} p^\mu_1 p^\nu_1 p^\rho_2 + G_{12} p^\mu_1 p^\nu_2 p^\rho_1 + G_{13} p^\mu_2 p^\nu_1 p^\rho_1 + G_{14} p^\mu_1 p^\nu_1 p^\rho_1 \]

with $\mathcal{M} \equiv \varepsilon^\nu_\rho(q) \varepsilon^\mu_{D^*}(p_1) \varepsilon^\rho_{D^*}(p_2) M^{\mu\nu\rho}(p_1^2, p_2^2, q^2) = \\
= \varepsilon^\nu_\rho(q) \varepsilon^\mu_{D^*}(p_1) \varepsilon^\rho_{D^*}(p_2) N_c \text{ tr } \int_\Lambda \frac{d^4k}{(2\pi)^4} \tilde{\Gamma}^\nu_{D^*}(k; -p_2) S_c(k + p_1) \Gamma^\mu_{D^*}(k; p_1) S_u(k) \tilde{\Gamma}^\rho_{\rho^0}(k; -q) S_u(k + q)$

- $\Gamma^\mu_{D^*}$: $D^*$-meson Bethe-Salpeter amplitude
- $\Gamma^\nu_{\rho^0}$: $\rho^0$-meson Bethe-Salpeter amplitude
- $\varepsilon^\mu_i$: Polarization vector
Why higher spin structures?

**Example:** decay of recently the discovered resonance $X(3872)$ ($J^{PC} = 1^{?+}$)

Main decay modes are $X(3872) \rightarrow D^0 D^{0*}(D^0 \pi^0), J/\psi \pi^+ \pi^-, J/\psi \gamma, J/\psi \pi^+ \pi^- \pi^+ ...$

✿ clear isospin violation
Exotic nuclear bound states: $J/\psi$ mass shift in nuclear matter

- $J/\Psi$ mass decrease in nuclear matter estimated to 4 – 7 MeV with QCD sum rules.
- An interesting challenge are properties of $D$ and $D^{(*)}$ mesons in medium:
  - Possible formation of $D(\bar{D})$ meson-nuclear bound states.
  - Enhanced dissociation of $J/\Psi$ in nuclear matter (heavy nuclei).
  - Enhancement of $D(\bar{D})$ production in antiproton-nucleus collisions.

$D$-meson interaction with nucleons

Antiproton annihilation on the deuteron (PANDA @ FAIR)

Meson exchange — effective Lagrangians
SU(4) symmetry used ....

SU(4) symmetry: \( g_{D\rho D} = g_{D\omega D} = g_{KK\rho} = \frac{1}{2} g_{\pi\pi\rho} \)

Jülich model:
A. Müller-Groeling et al. NPA 513, 557 (1990)
M. Hoffmann et al. NPA 593, 341 (1995)
**PS → S decays and rescattering**

Important in Dalitz plot analyses of non-leptonic three-body $D$ and $B$ decays

\[
\begin{align*}
D_{(s)} & \rightarrow f_0(980) & D_{(s)} & \rightarrow (\pi\pi)_S (KK)_S \\
D_{(s)} & \rightarrow K^*_0(1430) & D_{(s)} & \rightarrow (K\pi)_S \\
B_{(s)} & \rightarrow f_0(980) & B_{(s)} & \rightarrow (\pi\pi)_S (\bar{K}K)_S \\
B_{(s)} & \rightarrow K^*_0(1430) & B_{(s)} & \rightarrow (K\pi)_S
\end{align*}
\]

- So far there are no BSAs for scalar mesons
- More modeling and assumptions involved
- Scalar mesons tend to be broad (background)

Dyson-Schwinger and Bethe-Salpeter framework
Dyson-Schwinger equations

- Well suited to Relativistic Quantum Field Theory
- Non-perturbative continuum approach to QCD
- Hadrons as composites of Quarks and Gluons
- Qualitative and quantitative importance for:
  - Dynamical Chiral Symmetry Breaking
  - Generation of fermion mass from nothing
  - Quark & Gluon Confinement
- Method yields Schwinger functions (Euclidean Green’s Functions) = Propagators
- Confinement can be related to the analytic properties of QCD’s Schwinger functions
• No assumption of heavy-quark symmetry is made.

• In particular, pseudoscalar and vector meson masses are not degenerate.

• Weak decay constants are not degenerate.

• Although impulse approximation and truncations are employed, \( \frac{\Lambda_{QCD}}{m_c} \) contributions are systematically included.
The large current-quark mass of the $b$ quark almost entirely suppresses momentum-dependent dressing, so that $M_b(p^2)$ is nearly constant on a substantial domain. This is true to a lesser extent for the $c$ quark.

$$(\hat{M}^E)^2 = \{ s | s + M^2(s) = 0 \}$$

Euclidean mass definition
Euclidean mass, sigma term

Define a single quantitative measure, namely the renormalization-point invariant ratio

$$\zeta_f := \frac{\sigma_f}{M_E^f}$$

with the constituent-quark $\sigma$ term:

$$\sigma_f := m_f(\zeta) \frac{\partial M_E^f}{\partial m_f(\zeta)}$$

The solutions of

$$\left(\hat{M}^E\right)^2 = \{s|s + M^2(s) = 0\}$$

in GeV:

<table>
<thead>
<tr>
<th>$f$</th>
<th>chiral</th>
<th>$u, d$</th>
<th>$s$</th>
<th>$c$</th>
<th>$b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{M}_E^f$</td>
<td>0.42</td>
<td>0.42</td>
<td>0.56</td>
<td>1.57</td>
<td>4.68</td>
</tr>
</tbody>
</table>

The ratio $\zeta_f$ thus quantifies the effect of explicit chiral symmetry breaking on the dressed-quark mass function compared with the sum of the effects of explicit and dynamical chiral symmetry breaking.

Reasonable approximation:

$$S_{Q=c,b} \approx \frac{1}{i\gamma \cdot p + \hat{M}_Q}$$

$$\frac{f}{\zeta_f}$$
Strong decays: \( D^* \rightarrow D\pi \)

### Coupling yields \( D^* \) width

\[
A(D^* \rightarrow D\pi) = \epsilon_{\mu}^{D^*}(p_{D^*})M^\mu(p_D^2, p_{D^*}^2) := \epsilon_{\mu}^{D^*}(p_{D^*})p_D^\mu g_{D^*D\pi}
\]

\[
M^\mu(p_D^2, p_{D^*}^2) = N_e \text{ tr } \int_\Lambda \frac{d^4k}{(2\pi)^4} \bar{\Gamma}_D(k; -P_D)S_c(k + P_{D^*})i\Gamma_{D^*}^\mu(k; P_{D^*})S_u(k)\bar{\Gamma}_\pi(k; -Q_\pi)S_u(k + Q_\pi)
\]

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<thead>
<tr>
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<th>CLEO</th>
<th>DSE</th>
<th>QCDSR</th>
<th>Lattice</th>
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<tr>
<td>( g_{D^*D\pi} )</td>
<td>17.9 ( \pm 0.3 ) ( \pm 1.9 )</td>
<td>16.5 ( \pm 2 )</td>
<td>14.0 ( \pm 1.5 )</td>
<td>20 ( \pm 2 )</td>
</tr>
</tbody>
</table>

B. E., M.A. Ivanov and C.D. Roberts (2012)
Strong decays: \( D^* \to D\pi \)

\[
A(D^* \to D\pi) = e_{\mu}^{D^*}(p_{D^*}) M_{\mu}(p_D, p_{D^*})
\]

\[
M_{\mu}(p_D, p_{D^*}) = N_c \text{ tr} \int_{\Lambda}^{\infty} \frac{d^4 k}{(2\pi)^4} \bar{\psi}(k, p_{D^*}) \gamma_{\mu} \psi(k, p_D)
\]

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<th>( g_{D^*D\pi} )</th>
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<td>( \Gamma )</td>
<td>17.9 \pm 0.3 \pm 1.9</td>
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<td>20 \pm 2</td>
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B. E., M.A. Ivanov and C.D. Roberts (2012)

Similarly: \( D_s^* \to DK \)

\[
g_{D_s^*DK} = 20^{+2.5}_{-1.7}
\]

B. E., M.A. Ivanov and C.D. Roberts (2012)
Strong decays: $B^* \rightarrow B\pi$ (analogy)

This amplitude can be used for $m^2_\pi \rightarrow 0$ to extract $\hat{g}$ at leading order in HMChPT:

$$\hat{g} = \frac{g_{B^*B\pi}}{2\sqrt{m_B m_{B^*}}} f_\pi$$

<table>
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<tr>
<th></th>
<th>DSE model</th>
<th>Lattice in static limit ($n_f = 2$)</th>
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<tr>
<td>$\hat{g}$</td>
<td>0.37 ± 0.04</td>
<td>0.44 ± 0.03$^{+0.07}_{-0.0}$</td>
</tr>
</tbody>
</table>

DSE: B. E., M.A. Ivanov and C.D. Roberts (2011)


The value obtained from $D^*$ decay is: $\hat{g}_c = 0.56^{+0.07}_{-0.03} \rightarrow \Lambda_{QCD}/m_c$ corrections important!
Define \( \zeta_\rho \) as:

\[
\zeta_\rho := \frac{g_{D\rho D}(q^2)}{g_{K\rho K}(q^2)}
\]

Ratio measures the effect of SU(4) breaking \( \approx 300\% \)

SU(3) breaking \( \approx 20\text{–}30\% \)

\[ g_{D\rho D} \neq g_{K\rho K} \neq \frac{1}{2} g_{\pi\pi} \rho \]

Consequences?

The integrated $DpD$ interaction is enhanced by about 40% compared with an $SU(4)$ prediction for the coupling/form factor.

Large value value for the interaction strength entails an enhanced cross section in $DN$ scattering ($l = 1$ cross section inflated by a factor 4–5).

Possible novel charmed resonances or bound states in nuclei?
Higher-spin couplings

\[ g_{\mu\rho D^*} / m_{D^*} \]

\[ q^2 \text{ [GeV}^2\text{]} \]
Higher-spin couplings

![Graph showing higher-spin couplings](image-url)

- $D^* \rho D^*$
- Graph with $g_{D^*D^*\rho}$ vs. $q^2$ [GeV$^2$] for different $G_i$ values.

Legend:
- $G_3$
- $G_6$
- $G_5$
- $G_1 = G_5$
Form factors are the single important source of uncertainties and much work is left to pin down theoretical uncertainties to a few percent in charm physics.

One aspect learned over and over again is: heavy-quark symmetry is not applicable in charmed mesons and $\Lambda_{\text{QCD}}/m_c$ corrections are not negligible.

Other (flavor) symmetries often invoked to simplify calculations are not justified.

DSE/BSA treatments of heavy mesons must go beyond the rainbow-ladder approximation.